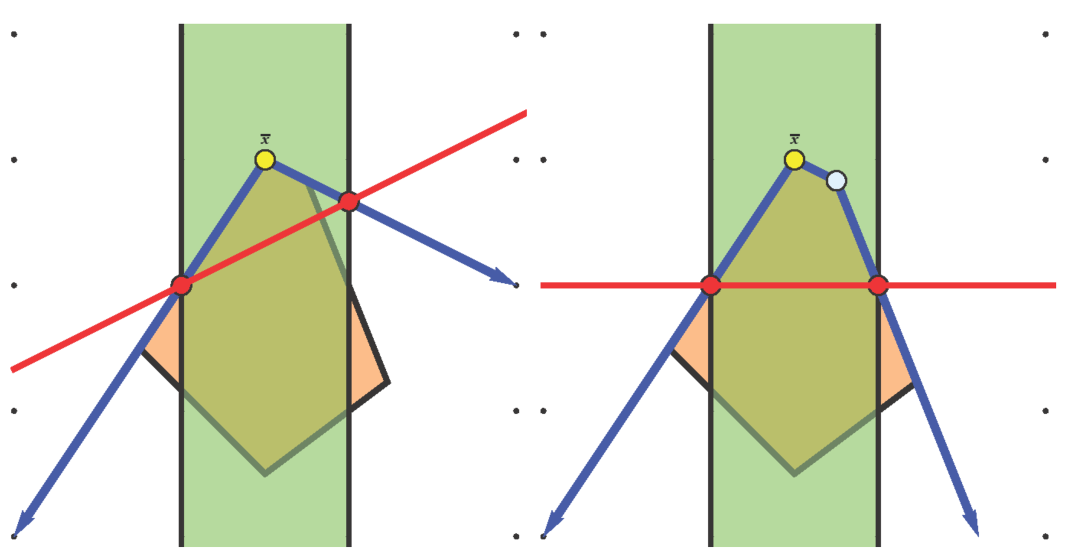
Computational investigation of generalized intersection cuts

Egon Balas, **Aleksandr M. Kazachkov**, François Margot, and Selvaprabu Nadarajah

Tepper School of Business, Carnegie Mellon University

INTRODUCTION



Input: MIP: $\min\{c^\mathsf{T}x : Ax \ge b, x \ge 0, x_j \in \mathbb{Z}, j \in I\}$

Notation:

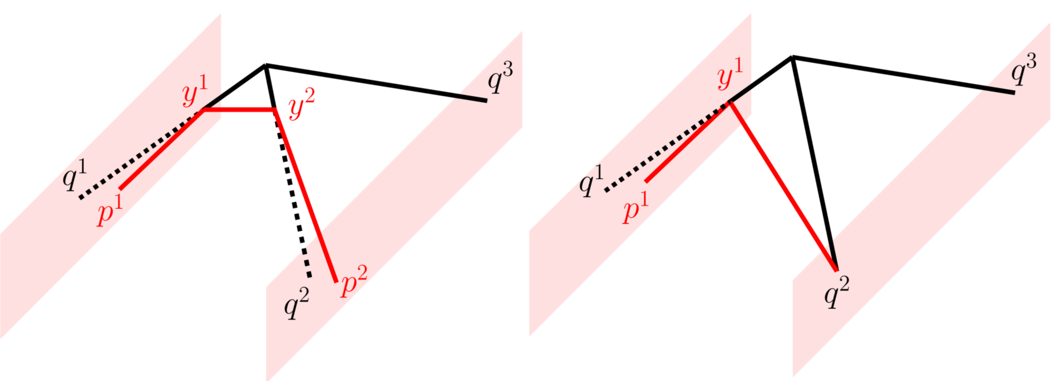
- $P := \{x : Ax \ge b, x \ge 0\}$
- $P_I := \{x \in P : x_j \in \mathbb{Z}, j \in I\}$
- $\bar{x} \in \operatorname{argmin}\{c^\mathsf{T}x : x \in P\}$
- $C(\bar{x})$: polyhedral cone obtained by taking all constraints corresponding to non-basic variables

Goal: A non-recursive method to generate valid cuts

Motivation: Avoid numerical issues encountered in standard recursive cutting plane procedures

Idea: Activate hyperplanes to obtain a tighter relaxation of P_I; full activation computationally expensive, hence *partial* hyperplane activation (PHA)

Full activation **Partial activation**



PHA$_{1,1}$: Intersect each ray of $C(\bar{x})$ with a hyperplane, activating it (partially) on that ray alone

Valid cuts: Consider the system, for $\beta \in \{-1,1\}$:

$$\alpha^\mathsf{T} p^j \ge \beta, \quad p^j \in \mathcal{P}$$
$$\alpha^\mathsf{T} r^j \ge 0, \quad r^j \in \mathcal{R}.$$

Here, $\mathcal{P}$ and $\mathcal{R}$ are points and rays generated by PHA$_{1,1}$. Any feasible solution $\bar{\alpha}$ with $\beta = \bar{\beta}$, such that $\bar{\alpha}^\mathsf{T}\bar{x} < \bar{\beta}$ yields a valid cut $\bar{\alpha}^\mathsf{T}x \ge \bar{\beta}$ for P_I.

Computational investigation: Experiment with various options for choosing hyperplanes in PHA$_{1,1}$, test effect of cutting rays by additional hyperplanes, and compare strength of cuts obtained from different objectives used with the cut LP

RESULTS

Experimental setup: Instances selected from MIPLIB 3 based on time taken to test one set of parameters. Compared generalized intersection cuts (GICs) to *standard* intersection cuts (SICs), which are known to be strong.

Hyperplane selection. Choose hyperplane that:

1. (HH1) Intersects ray first
2. (HH2) Gives intersection points with best average depth
3. (HH3) Creates largest number of *final* intersection points (final means the point is in P)

Cut selection.

1. Cut LP with objective:
   i. (N) Ray directions of $C(\bar{x})$
   ii. (C) Vertices v^{jh} created during PHA
   iii. (S) Intersection points from other splits
2. (B) Solve a bilinear program:

$$\min_{\alpha,x} \ \alpha^\mathsf{T}x$$
$$\alpha^\mathsf{T}p^j \ge \beta, \quad p^j \in \mathcal{P}$$
$$\alpha^\mathsf{T}r^j \ge 0, \quad r^j \in \mathcal{R}$$
$$x \in \overline{P}$$

Number of hyperplanes cutting a ray: Tested effect of activating up to three hyperplanes per ray (+1H, +2H, +3H). First hyperplane selected by one of rules above; additional ones activated to maximize number of final intersection points.

In the (separable) bilinear program, $\overline{P}$ refers to P intersected with all the standard intersection cuts. It is solved iteratively over each of the variable sets, which only appear together in the objective.

Table 1: Percentage gap closed by hyperplane activation procedure

	SIC	GIC	GIC-SIC	HH1	HH2	HH3	+1H	**+2H**	+3H
bell3a	59.13	63.17	4.04	2.60	3.46	4.04	2.54	**3.46**	4.04
bell4	23.37	26.47	3.10	1.85	3.10	1.71	2.84	**3.10**	2.38
bm23	5.92	9.61	3.68	3.01	3.68	3.68	2.79	**2.91**	3.68
egout	53.77	54.50	0.73	0.64	0.73	0.64	0.73	**0.02**	0.00
gt2	58.36	77.70	19.34	19.34	19.34	19.34	3.19	**19.34**	12.47
lseu	4.36	4.67	0.31	0.26	0.31	0.31	0.01	**0.01**	0.31
misc02	0.00	2.36	2.36	1.54	0.00	2.36	0.00	**1.89**	2.36
misc05	4.20	4.20	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00
mod013	4.41	9.99	5.58	5.58	4.95	5.58	4.95	**5.58**	4.95
p0033	1.86	2.62	0.76	0.76	0.76	0.76	0.76	**0.00**	0.76
p0201	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00
pipex	0.82	1.45	0.63	0.63	0.63	0.63	0.63	**0.63**	0.04
sample2	3.91	21.09	17.19	17.19	17.19	17.19	17.19	**17.19**	0.00
sentoy	10.38	13.25	2.87	2.64	2.61	2.87	1.77	**2.87**	2.61
stein15_nosym	50.00	58.00	8.00	8.00	8.00	4.06	8.00	**8.00**	4.06
Average	18.70	23.27	4.57	4.27	4.32	4.21	3.03	**4.33**	2.51

Table 2: Percentage gap closed by objectives used for cut LP

	All	N	N+B	N+S	N+C	N+B+S	N+B+C	N+S+C
bell3a	**4.04**	0.00	4.04	3.06	4.04	4.04	4.04	4.04
bell4	**3.10**	0.00	1.47	2.66	3.10	3.04	3.10	3.10
bm23	**3.68**	0.00	2.59	3.68	3.68	3.68	3.68	3.68
egout	**0.73**	0.00	0.29	0.02	0.73	0.29	0.73	0.73
gt2	**19.34**	0.00	4.61	0.00	19.34	13.31	19.34	19.34
lseu	**0.31**	0.00	0.01	0.17	0.31	0.17	0.31	0.31
misc02	**2.36**	0.00	0.00	2.36	2.36	2.36	2.36	2.36
misc05	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
mod013	**5.58**	0.00	0.31	4.95	5.58	4.95	5.58	5.58
p0033	**0.76**	0.00	0.76	0.76	0.76	0.76	0.76	0.76
p0201	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pipex	**0.63**	0.00	0.04	0.63	0.04	0.63	0.04	0.63
sample2	**17.19**	0.00	0.00	15.62	17.19	15.62	17.19	17.19
sentoy	**2.87**	0.00	2.60	0.97	2.87	2.61	2.87	2.87
stein15_nosym	**8.00**	0.00	8.00	8.00	8.00	8.00	8.00	8.00
Average	**4.57**	0.00	1.65	2.86	4.53	3.96	4.53	4.57

CONCLUSION

Activating an additional hyperplane per split increases the strength of cuts. However, activating a third hyperplane sometimes leads to worse cuts.

Investigation showed that strategy C performs nearly as well as more sophisticated methods (S, B).

Future research will aim to address the questions:

1. Why are there so few GICs generated?

Table 3: Maximum number GICs generated

	Splits (SICs)	Max GICs	Active GICs	Obj tried	SIC opt in cut LP
bell3a	32	95	23	6784	2448
bell4	46	164	42	6326	4542
bm23	6	30	5	404	110
egout	40	31	26	2350	2218
gt2	11	27	8	3165	2969
lseu	11	55	11	3993	1366
misc02	14	70	10	2589	1040
misc05	11	55	23	9663	7260
mod013	5	25	7	461	194
p0033	7	12	4	299	165
p0201	20	46	35	2653	1680
pipex	6	30	17	743	281
sample2	9	29	8	1016	696
sentoy	8	40	7	679	251
stein15_nosym	5	25	14	88	9

2. Why does the third hyperplane, while adding more deep and final points, lead to worse cuts?

Table 4: Number points generated

	Points			Final points		
	+1H	+2H	+3H	+1H	+2H	+3H
bell3a	8850	9317	9274	1448	1448	1448
bell4	4515	4386	4165	1029	1035	1037
bm23	2492	2527	2795	342	359	371
egout	440	389	406	93	93	93
gt2	5512	4005	3404	495	495	496
lseu	18081	17351	16560	2046	2062	2107
misc02	7271	7219	7599	189	227	306
misc05	17184	16815	16090	274	274	286
mod013	1816	1730	1747	196	196	196
p0033	213	228	259	92	95	95
p0201	4029	5721	6928	32	75	95
pipex	1408	1550	1597	507	527	531
sample2	765	720	694	103	103	103
sentoy	17915	19438	21123	2530	2683	2809
stein15_nosym	285	272	268	2	2	2
Average	6052	6111	6194	625	645	665

3. How do we identify good objectives to use in the cut LP?

4. What is the effect of using other cut generating sets such as triangles and parametric octahedra?

REFERENCES

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